

# The Credit Risk in Stock-Based Loans

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A dissertation submitted to the Faculty of Commerce, University of Cape Town, in partial fulfilment of the requirements for the degree of Master of Philosophy.

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*MPhil in Mathematical Finance,  
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# Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy to the University of Cape Town. It has not been submitted before for any degree or examination to any other University.

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July 26, 2018

# Abstract

Stock-based loans are an increasingly popular form of loan that are collateralised using stocks. Since these loans are often non-recourse loans, the lenders are subject to the risk that the collateral is worth less than the loan, and the borrower defaults. This dissertation will consider the credit risk faced by lenders when issuing these loans. To achieve this, this dissertation will propose different models to quantify this risk using various credit measures. A sensitivity analysis to key model parameters is then conducted. Some brief comments about capital requirements will also be made.

# Acknowledgements

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## Chapter 1

# Introduction

A stock-based loan (SBL) is a loan where a stock or portfolio of stocks is used as collateral. Since these loans are typically non-recourse, the lender is subject to the risk that the stock value at maturity is lower than that owed by the borrower, in which case the borrower has no incentive to repay the loan and thus defaults.

When default occurs, the lender suffers a loss equal to the difference between the value of the loan and the value that can be realised for the stock-collateral. Typically, the stock-collateral is a multiple of the loan value at inception of the SBL and the share price would need to decline significantly over the maturity of the loan for the lender to suffer a loss.

However, the lender is often required by regulation to hold capital to protect against this possibility. Such regulatory capital comes at a cost. This cost is borne by the lender, and passed on to the borrower through a higher rate of interest on the loan.

Adequately modelling the SBL and its associated probabilities of default and expected losses, amongst other credit measures, will allow the lender to more accurately determine the amount of capital to hold. By avoiding the need to be overly prudent, the lender could reduce the cost of issuing these loans and thus offer a more competitive product.

This dissertation will examine SBLs from the lenders perspective and will analyse the credit default risk in these products.

In Chapter 2, SBLs are described in detail and relevant terminology is introduced and defined. A brief literature review is also provided.

Chapter 3 considers the problem of modelling the SBL. The basic modelling framework is described and the necessary notation is introduced. Models of the share price are proposed along with the reasoning for why these models are expected to be able to calculate and model the risk adequately. A brief description of the data used in the investigation is provided. The estimation of the model parameters from the data is also considered.

The resulting credit-risk measures from the modelling investigation are presented in Chapter 4. An analysis is then conducted into the effect that the SBL specifications and modelling parameters have on the risk measures.

Chapter 5 concludes the dissertation. The capital requirements of the SBL are briefly discussed and possible avenues of future research are considered.

## Chapter 2

# Stock-based loans

### 2.1 Description of stock-based loans

A stock-based loan (SBL) is a loan where a stock or portfolio of stocks is used as collateral. This would be useful, for example, when the borrower has a large portfolio of shares, but is in need of cash. He could then receive a loan from a lender, and give his portfolio to the lender as security on the loan ([Xia and Zhou, 2007](#)). This dissertation will primarily focus on single-stock portfolios.

The loan accrues interest at the sum of a reference rate and some spread. Generally, the dividends from the equity portfolio pay back the loan during the duration of the contract. At maturity of the loan, the borrower will pay off the loan and regain ownership of his shares. The loans generally have maturities of between two and six years ([Backwell \*et al.\*, 2016](#)). If desired, the contract can be renewed at maturity. It will be assumed that early repayment of the loan is disallowed.

The ratio of the value of the shares to the value of the loan is called the share coverage ratio. The first leg of the transaction occurs at an initial share coverage ratio (*ICR*) which is typically around a value of three. To make the loan attractive, lenders will try to keep the *ICR* as low as possible, given their risk considerations. A minimum share coverage ratio (*MCR*), usually in the region of two, is also specified ([Backwell \*et al.\*, 2016](#)).

Additionally, SBLs usually give the borrower the option to not repay the loan. This action is only economically rational if the value of the collateral were to decrease below the value of the loan. As a measure of protection, the lender will not allow the share coverage ratio to decrease below the minimum share coverage ratio; if it does, then a *breach* is said to have occurred. The lender can then, at its discretion, decide to liquidate the share portfolio to recover the outstanding loan amount. Any excess of the liquidation value over the loan is handed over to the borrower. The dissertation will only consider SBLs that have this optionality feature.

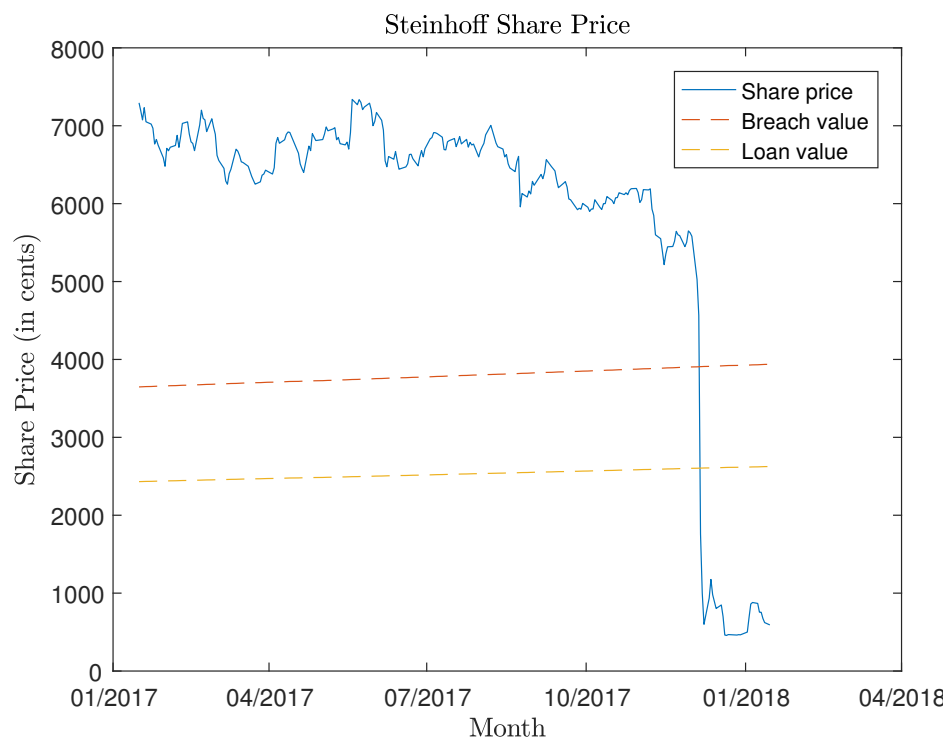
The loans are non-recourse in that if the lender fails to recover the full loan va-

lue upon liquidating the equity portfolio, the borrower is not liable, and the lender suffers a loss (Xia and Zhou, 2007). If the loan was not non-recourse, then on maturity of the loan, if the collateral is worth less than the amount owed, the borrower will have less of an incentive to default. If the borrower, however, does decide to default, the lender can still try to recover his capital through bankruptcy proceedings. By forgoing this protection, the lender makes the SBL arrangement more attractive to the borrower, and can charge a higher rate of interest.

The advantage to the borrower of using such a loan is that he is able to avoid selling his shares and can thus avoid capital gains tax, transaction costs and the risk of having to buy the shares later at a higher price. He is also charged a lower rate of interest as a result of providing collateral. Another benefit is that the stock loan provides a hedge against a large drop in the value of the share portfolio over the term of the loan (Xia and Zhou, 2007). The use of stock-based loans also allows the borrower to diversify his overall portfolio by using the proceeds from the loan to invest in different asset classes.

The advantage to the lender is that, since equities are generally liquid, the lender can expect to sell off the collateral relatively quickly. Furthermore, as a result of the use of such collateral, the credit risk they face is diminished and they will thus be required to hold less capital. The lower credit risk also means that lenders can avoid extensive due diligence (Backwell *et al.*, 2016).

Of relevance to this dissertation is the example of the retailer, Steinhoff International Holdings. Banks had provided funding of about R22 billion, backed against Steinhoff shares, to an investment firm controlled by the chairman of Steinhoff (Hyuga, 2018). When Steinhoff then publicly revealed in early December 2017 that they had uncovered accounting irregularities, the share price lost more than 80% of its value in one day (see Figure 2.1). Four US banks alone are reported to have suffered more than R12 billion in mark-to-market losses and charge-offs in loans related to Steinhoff (Keller and Campbell, 2018). Now, for example, if the banks had issued the loans at the beginning of 2017 at an initial coverage ratio of 3, the loan value per share used as collateral would have been around 2400 cents, as can be seen in Figure 2.1. It is assumed here that the loans were issued at an interest rate of 8% and are not repaid during the year. The corresponding breach value (using a minimum coverage ratio of 1.5) is also shown in the figure. The precipitous drop in the share price will lead to both breach and loan breach occurring immediately and the lender incurring losses.



**Fig. 2.1:** The share price of Steinhoff over the period from January 2017 to January 2018. Example loan and breach values are provided.

## 2.2 Literature review

The first model for the stock-based loan was proposed by [Xia and Zhou \(2007\)](#) who primarily try to establish the explicit value of the loan under the Black–Scholes–Merton model. As such, they employ the risk-neutral measure,  $\mathbb{Q}$ , and assume the share price follows a geometric Brownian motion (GBM). The share is assumed to have a continuous dividend yield. They then translate the problem of valuing the stock-loan into that of evaluating an American call option with a strike price dependent on time. The product they analyse allows for early repayment, unlike the SBLs considered in this dissertation. They also assume, for mathematical tractability, that the SBL has an infinite life.

[Dai and Xu \(2011\)](#) treat the valuation of the stock loan as an optimal stopping problem. Unlike [Xia and Zhou \(2007\)](#), they are concerned with finite-maturity stock loans. They also consider various ways of dividend distribution as they state it has a bearing on the optimal strategy of the borrower: the dividends could be gained by the lender prior to redemption, the dividends could be returned to the borrower during the lifetime of the SBL or the reinvested dividends could be returned to the borrower on redemption.

[Zhang and Zhou \(2009\)](#) value the SBL under a regime-switching model where the share price follows a GBM. They also proceed to derive equations to value the loan where they use an approach involving variational inequalities.

[Cai and Sun \(2014\)](#) consider both infinite- and finite-maturity SBLs under a hyper-exponential jump diffusion model. For the SBL with an infinite-maturity, they derive closed-form formulae for the price of the SBL and provide analytical approximations in the finite-maturity case. Their choice of the hyper-exponential model is justified by it being better than the Black–Scholes–Merton model at representing the leptokurtic nature of stock-returns, and due to the analytical solutions it provides for some path-dependent options.

[Grasselli and Gómez \(2013\)](#) also look at the stock loan from the borrower's point of view, but note that, in reality, the borrower may not be able to find the unique arbitrage-free price for the loan. They thus extend the stock loan valuation to incomplete markets.

A report prepared by [Backwell \*et al.\* \(2016\)](#) considered the credit risk in stock-based lending. The report analysed various models and computed various credit measures and sensitivities. This dissertation will follow a similar approach to the report.

## Chapter 3

# Modelling stock-based loans

### 3.1 The basic framework

Suppose that, at time 0, an amount  $\bar{N}_0$  is loaned with  $n_s$  shares being placed as collateral under the loan agreement. Let  $S_t$  be the value of the share at time  $t$ . Then, the initial share coverage ratio is

$$ICR = \frac{n_s S_0}{\bar{N}_0}.$$

This dissertation will consider the normalised value  $N_t = \frac{\bar{N}_t}{n_s}$ , and will simply refer to it as the loan value.

The cumulative dividend process is represented by  $D_t$ . Suppose that interest is charged on the loan at the total of the risk free rate,  $r$ , and a spread,  $s$ . The loan process then has the dynamics

$$dN_t = (r + s)N_t dt - dD_t.$$

This has the interpretation that the loan continually accrues interest at a rate equal to the sum of the risk-free rate and the spread. The loan is paid down by dividend payments from the share.

The time of breach is the stopping time

$$\tau = \inf \left\{ t \geq 0 \mid \frac{S_t}{N_t} \leq MCR \right\}.$$

The breach level is equal to the product of the loan value and  $MCR$ . Thus, breach occurs at the earliest time at which the share price decreases below this breach level. The event where the share price,  $S_t$ , falls below the loan value,  $N_t$ , is defined as *loan breach*.

The loan is defined to be in default when the lender starts making losses on the SBL. This happens when the share price falls below the loan value to such an extent that the sale of the collateral is insufficient to recover the loan amount. Note that

since the breach barrier is set to a multiple of the loan value, the share price will be greater than the loan value initially. Default occurs when these initial excesses  $(S_t - N_t)$  are not enough to offset the losses made if loan breach occurs during the liquidation period.

Typically, when breach occurs, the lender will consider various factors before deciding whether to liquidate the shares. But here, for the sake of simplicity, it will be assumed that the lender will always liquidate. This assumption is not expected to have a great impact on results, given that the liquidation process starts at a multiple of the amount owed.

It will also be assumed that, upon breach, the lender will liquidate the shares over  $n_d \in \mathbb{Z}^+$  days to avoid adversely affecting the market price of the shares. However, spreading the sale over too long a period will increase the likelihood of losses given the possibility of further drops in the share price. Assuming 250 trading days in a year, the liquidation value, as specified in [Backwell et al. \(2016\)](#), is

$$X_\tau = \sum_{k=0}^{n_d-1} \frac{1}{n_d} e^{-rk/250} S_{\tau+k/250}.$$

This is the discounted value, at the time of breach, of the shares sold over the following  $n_d$  days. Having to sell the share following breach, even if the sale is spread over a few days, could still adversely affect the share price. To better account for this impact, the dissertation will include a *liquidity pressure* term,  $l \in \mathbb{R}_{(0,100]}$ . For each day of trying to sell the share, the market price is forced downwards by  $l\%$ . The liquidation value is then

$$X_\tau = \sum_{k=0}^{n_d-1} \frac{1}{n_d} e^{-rk/250} S_{\tau+k/250} (1 - l\%)^k.$$

The variable  $l$  is typically dependent on the size of the contract and the liquidity of the collateral, but for this investigation will be assumed to be around 1%. The liquidity pressure term is expected to increase the likelihood and magnitude of losses.

The lender will need to analyse the profit and risk characteristics of the loan arrangement. The net present value (NPV) of the SBL to the lender is then

$$NPV = -N_0 + \int_0^{\min(\tau, T)} e^{-ru} dD_u + I_{\tau < T} [\min(N_\tau, X_\tau) e^{-r\tau}] + I_{\tau \geq T} [N_T e^{-rT}].$$

This is the sum of the present value of the dividends until the earlier of breach or maturity and a term contingent on whether breach occurs, netted off against the initial loan value. If breach does occur, the lender receives the discounted value of



the smaller of the liquidation value or the loan value at the time of breach. This is because any proceeds from liquidation exceeding the loan value are due to the borrower. If breach does not occur, then the lender simply receives the discounted value of the loan at maturity. Thus, note that the  $NPV$  defined here is a random variable.

If the  $NPV$  is positive, the lender makes a profit. The distribution of the  $NPVs$  will give an indication of the range of possible profit (or loss) scenarios. The expected net present value statistic is simply

$$ENPV = \mathbb{E}[NPV].$$

The distribution of  $NPVs$  can also be used to determine the Value at Risk (VaR). The VaR figure is the maximum loss that can be suffered at a specified probability level,  $p$  (Hull, 2012). That is,

$$VaR = -\inf\{x \in \mathbb{R} \mid \mathbb{P}[NPV \leq x] \geq p\}.$$

The probability of breach over the term of the SBL is denoted by  $PB = \mathbb{P}[\tau < T]$ . The loss given breach is the average loss arising from failing to recover the loan value once breach has occurred. It is defined as

$$LGB = \mathbb{E}[\max(N_\tau - X_\tau, 0) \mid \tau < T].$$

The probability of default is defined as

$$\begin{aligned} PD &= \mathbb{P}[\max(N_\tau - X_\tau, 0) > 0] \\ &= \mathbb{P}[N_\tau > X_\tau]. \end{aligned}$$

The loss given default can be defined as

$$\begin{aligned} LGD &= \mathbb{E}[\max(N_\tau - X_\tau, 0) \mid \text{default occurs}] \\ &= \frac{LGB \cdot PB}{PD}. \end{aligned}$$

The expected loss (EL) is given by

$$\begin{aligned} EL &= PB \times LGB \\ &= PD \times LGD. \end{aligned}$$

## 3.2 Modelling the share price process

When modelling the SBL, the share price is the key concern. Recall that breach is when the share price dips below the product of the loan value and the minimum

share coverage ratio. Furthermore, loan breach is when the share drops below the loan value.

There are thus three classes of share price paths that effectively determine the value of the loan.

1. Paths that do not result in breach.
2. Paths that result in breach, but not in loan breach.
3. Paths that result in loan breach.

The first two cases are generally not cause for concern as in both cases, the lender will be able to liquidate the shares at a good price and recoup the loan value. It is the third case that determines the extent of the losses that the lender will suffer. It is thus crucial that whichever model is used, the probability of loan breach occurring is adequately realistic so that paths of the third type do occur sufficiently often.

Furthermore, both cases 1 and 2 do not materially affect the net present value or loss given breach of the SBL. Therefore, the dynamics of the share price in those scenarios where loan breach does not occur is irrelevant as the lender will almost surely be able to recover his capital. The dynamics of the share price, once loan breach has occurred, is of more importance as it determines the actual recovery from liquidation.

Credit risk measures need to be calculated under the real-world measure,  $\mathbb{P}$ , for them to be readily interpreted by the lender and used for risk management purposes. Hence, the share price, and thus the SBL, will be modelled under  $\mathbb{P}$  in the analysis that follows.

### Modelling assumptions

It will be assumed that there are no transaction costs and that there are no capital gains and dividends taxes.

There are a number of choices available for modelling dividends. These include assuming proportional dividends using a continuous dividend yield or a discrete dividend yield, variations where the yield is stochastic or using fixed dividends (which are not dependent on the share price on the dividend date). [Haug \*et al.\* \(2003\)](#) state that it is important to model the dividends from a stock discretely when the claim is contingent on one stock. If the stock-collateral consists of a large portfolio of stocks then the discrete payouts can be approximated with a continuous dividend yield. This dissertation mainly focuses on single-stock portfolios, and thus will employ a discrete dividend yield throughout.

Further simplifying assumptions were made regarding dividends. Dividends will be paid every 125 business days. The ex-dividend date and the payment date are assumed to be the same. It is assumed that the dividend at dividend date  $t$  is equal to  $y/2 \times S_t$ , where the constant  $y$  is the discrete dividend yield. This, in effect, means that the share price and dividend are assumed to be perfectly correlated. Also, the first dividend will only be paid half a year from the inception of the contract. The value of the dividend is immediately subtracted from the share price (and company value, for the structural models), as well as the loan value. This is another simplifying assumption as the drop in the share price on the ex-dividend date is typically less than the value of the dividend, even when there are no taxes on the dividends (Frank and Jagannathan, 1998). For the sake of simplicity, in the model descriptions that follow, the dividends process will not be included in the share price dynamics.

The risk of liquidation under distress is an important consideration given its impact on the risk and profitability of the contract. Trying to liquidate the shares in a market where the share price is in rapid decline can subject the lender to significant losses. Thus it is assumed that the lender will sell the shares over a liquidation period of 10 days. Furthermore, as mentioned previously, the share price is depressed by a fixed percentage for each day of the liquidation period.

It is also assumed that the whole portfolio of shares used as collateral will be sold during the liquidation period. In reality, the lender will only sell as many stocks as is needed to recover the loan amount, with the remaining shares being returned to the borrower. However, for the purposes of this dissertation, this assumption, which simplifies the modelling, is not expected to have a significant effect on the financial position of the lender since any excess of the liquidation value would be returned to the borrower anyway.

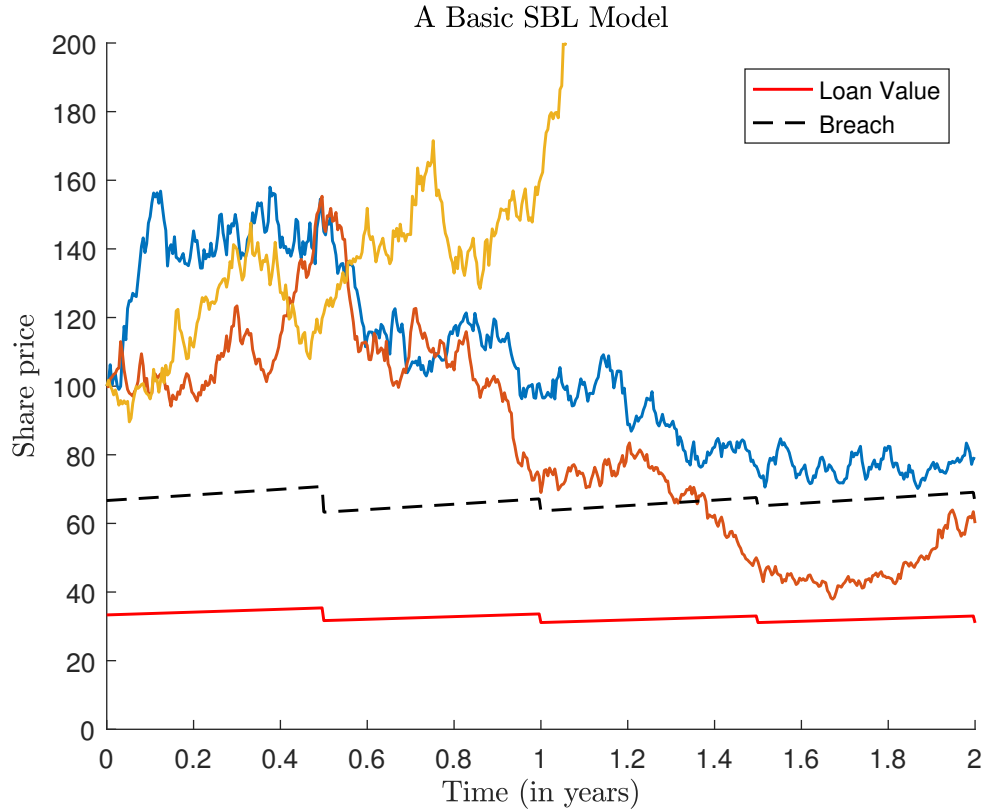
Interest rates will be assumed to be constant and independent of the share price. If in reality the interest rate is negatively correlated with the share price, for example, then an increase in the interest rate is expected to lead to a decrease in the share price. This, along with the subsequent larger increase in the breach value, will mean that the probability of breach is expected to be higher.

Thus, this assumption is only appropriate for stocks that are approximately independent of the interest rate. Flannery and James (1984) state that changes in the interest rate are significantly correlated to movements in the stock price of commercial banks and savings and loan associations. Therefore it might not be appropriate to use the models considered in this dissertation when the stock-collateral predominantly consists of these types of shares.

### GBM share price

For a diversified portfolio consisting of a large number of shares, the total value of the portfolio can be modelled adequately with a geometric Brownian motion (GBM) process. A simple example of such a portfolio would be an index such as the JSE Top40.

However, for a portfolio consisting predominantly of one share, using the GBM model for the share price in SBLs would be inappropriate. The primary issue with this is that, for a share that breaches, the probability of the share price going below the loan value during the short liquidation period is unrealistically low — see Figure 3.1 for example. This will lead us to underestimate the expected loss from a breach. This low probability is not as unrealistic for a well-diversified portfolio.



**Fig. 3.1:** A simulation of an SBL where the shares are modelled with GBMs. The loan and breach values correspond to the share plotted in red. Note that a liquidation period of 10 days amounts to about 0.03 years.

The share price  $S_t$  is simply modelled under the real world measure,  $\mathbb{P}$ , with the

dynamics

$$dS_t/S_t = \mu dt + \sigma dW_t,$$

where  $\mu \in \mathbb{R}$  and  $\sigma \in \mathbb{R}^+$  are the drift and volatility of the share price process.

The simplicity of this model means that only two parameters need to be estimated: the drift  $\mu$  and volatility parameter  $\sigma$  can be estimated from empirical data as the mean and standard deviation of the daily log-returns.

The share price paths can be simulated in steps of  $\Delta t = \frac{1}{250}$  for  $t \leq T$  using

$$S_t = S_{t-\Delta t} e^{(\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}Z},$$

where  $Z$  is a standard normal random variable.

### GBM share price with jump to distress

One of the key assumptions of the Black–Scholes–Merton model is that the stock price process has continuous sample paths with normally distributed log-returns. Thus, the stock price, modelled by a GBM, has a log-normal distribution between any two time points. However, empirical studies indicate that in the stock price time series there are more outliers than what would be predicted by this distribution. This suggests that there could be jumps in the share price process (Merton, 1976).

To better model defaults of the loan in this model, a jump to distress term can be added. This term represents an unforeseen, extreme downward move in the share price. With a jump to distress, jumps are assumed to randomly occur at a Poisson rate of  $\lambda \in \mathbb{R}^+$ . Once the jump occurs, the share price drops to a fraction of its price at the previous instant in time. That is,  $S_t = RS_{t-}$ , where  $R \in \mathbb{R}_{[0,1]}$ . The dissertation will assume that  $R \sim U(0, 0.8)$ , so that once the jump to distress occurs, we observe at least a 20% drop in price. The choice of the uniform distribution is simplistic; a more empirically justified distribution is left for further investigation.

Given a large enough dataset consisting of many stocks over a long period of time, the probability of a jump to distress occurring in any one year can be estimated from this historical data. Since such a dataset was not available, the probability of jumping to distress in any given year is exogenously chosen to be 0.3%. This will give a roughly 1% chance of a jump to distress occurring over the 3-year period. This is a very simple way of allowing the model to better capture the risk of default, however, this model can be criticised for being unrealistic as it assumes that it is equally likely for a well-performing share to abruptly jump to distress as it is for a poorly-performing share.

### Empirical model

The empirical model uses the actual price history of a stock or index to generate share price paths.

Empirical log-returns are usually negatively skewed as the magnitudes of the negative jumps are typically greater than the positive jumps. This would result in a negative coefficient of skew (Hanson and Westman, 2002).

The coefficient of kurtosis of the normal distribution is 3, however the kurtosis of the empirical log-returns distribution is typically larger than 3, meaning that it is leptokurtic (Hanson and Westman, 2002). This effectively means that the probability distribution has fatter tails and a thinner peak.

To generate share price paths, the log-returns are calculated from the price data and then its mean  $\mu$  and standard deviation  $\sigma$  are computed. Then, the log-returns are standardised using  $\mu$  and  $\sigma$ . Denote by  $Z_E$  the random variable generated by sampling from this set with replacement. The share price at time  $T$  can then be simulated using,

$$S_T = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z_E}.$$

### The Merton jump model

Merton (1976) extended the Black–Scholes–Merton model to include jumps. In the Merton jump model, under the real-world measure  $\mathbb{P}$ , the stock price satisfies

$$\frac{dS_t}{S_{t-}} = \mu dt + \sigma dW_t + (J_t - 1)dN_t, \quad (3.1)$$

where  $\mu$  is the drift rate,  $\sigma$  is the volatility of the share price process,  $\{W_t\}$  is a standard Brownian motion process and  $\{N_t\}$  is a Poisson process with jump intensity  $\lambda$ . The size of the jump satisfies  $\log J_t \sim N(\alpha, \beta)$  for  $\alpha \in \mathbb{R}, \beta \in \mathbb{R}^+$  (Backwell *et al.*, 2016). The jump diffusion parameters  $r, \sigma, \lambda, \alpha, \beta$  are all assumed to be constant. The Brownian motion process and the Poisson process are pairwise independent. The jump amplitudes are independent random variables. The estimation of the jump diffusion parameters is done in Section 3.4.

Given adapted coefficients  $b_t, \sigma_t$  and a predictable function  $\delta_t(\cdot)$ , consider the one-dimensional Itô process  $X$  that, for  $t \geq 0$ , obeys the following dynamics:

$$dX_t = b_t dt + \sigma_t dW_t + \delta_t(J_t)dN_t. \quad (3.2)$$

Suppose that  $u = u(t, x)$  is a real valued function that is twice-continuously differentiable for  $t \geq 0$ . Then Itô's lemma states that

$$\begin{aligned} du(t, X_t) = & \left( \partial_t u(t, X_t) + \partial_x u(t, X_t)b_t + \frac{1}{2}\partial_{xx}^2 u(t, X_t)\sigma_t^2 \right) dt + \partial_x u(t, X_t)\sigma_t dW_t \\ & + [u(X_{t-} + \delta_t(J_t)) - u(X_{t-})] dN_t. \end{aligned}$$

Under the risk-neutral measure  $\mathbb{Q}$ , Crépey (2013) specifies the share price dynamics as

$$\frac{dS_t}{S_{t-}} = rdt + \sigma dW_t + (J_t - 1)dN_t - \lambda \bar{J}dt,$$

where  $\bar{J} = \mathbb{E}[J_t - 1] = e^{\alpha+\beta/2} - 1$ . This suggests that in the Itô process (3.2),  $b_t = S_{t-}(r - \lambda \bar{J})$ ,  $\sigma_t = S_{t-}\sigma$ , and that  $\delta_t(J_t) = S_{t-}(J_t - 1)$ .

Using Itô's lemma, the log-stock  $\log(S_t)$  can be written as

$$d \log(S_t) = (r - \frac{1}{2}\sigma^2 - \lambda \bar{J})dt + \sigma dW_t + \log(J_t)dN_t.$$

Solving this SDE yields a formula for simulating  $S_T$  given a starting value  $S_0$ ,

$$S_T = S_0 e^{(r - \frac{1}{2}\sigma^2 - \lambda \bar{J})T + \sigma W_T} \prod_{i=1}^{N_T} J_i.$$

Note that under the real-world measure  $\mathbb{P}$ , the  $\lambda \bar{J}$  term would be absent in the above formula.

### The Merton jumps plus model

Using the estimated parameters for jumps and volatility allows us to model the probability of breach adequately. However, continuing the use of these parameters after breach will not necessarily allow for the probability of sharp decreases within the liquidation period. This increased volatility could be as a result of the leverage effect or because the stock could be in distress. Once breach has occurred, different volatility and jump parameters could be used to portray the increased volatility and to get default levels more reflective of reality. This, although artificial, does allow us to better model the risks in the share-based loan.

This approach was implemented for the Merton jump model where the jump intensity,  $\lambda$ , and the volatility of the GBM process,  $\sigma$ , are doubled upon breach. The choice to double was arbitrary and can be changed as appropriate. To make referring to this model in the subsequent sections easier, this model is termed the *jumps plus model*.

### The Merton jumps plus model with jump to distress

A jump to distress term, like the one added to the GBM share price process earlier, is also added to the Merton jumps plus model. The probability of jumping to distress is taken to be the same as in the GBM model.

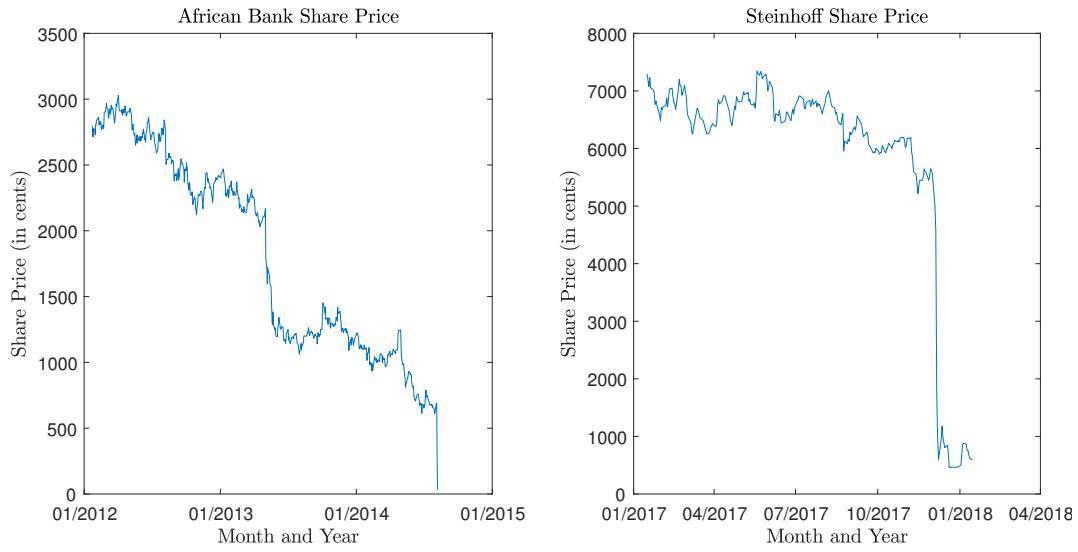


Fig. 3.2: The stock price paths of the shares used in the distress distribution.

### Jump to an empirical distress distribution

Regime-switching models allow for the stock to randomly switch to another distribution, say a distress distribution, which could represent the returns of a stock in distress. These models are similar to the *jumps plus* model used in this dissertation.

This dissertation will assume a distress distribution that is a mixture of the Steinhoff and African Bank datasets (see Figure 3.2). Two datasets are used to get a better representation of a share in distress.

One benefit of using an empirical distribution is that this distribution implies the behaviour under distress of the share, and thus this behaviour does not have to be assumed, as had to be done with the jump to distress term mentioned before. The probability with which a jump to the empirical distress distribution occurs in any given year will be assumed to be 1%. Ideally, this could be estimated from historical returns. This is left for further study.

### The Merton credit model

In the Merton (1974) credit model, the equity  $E_t$  is modelled as a call option on the value of the assets,  $V_t$ , with a strike price equal to the amount of debt,  $D$ . The share price,  $S_t$ , is equal to  $\frac{E_t}{n_s}$ , where  $n_s$  is the number of shares in issue.

The firm's asset value  $V_t$  can be modelled, under  $\mathbb{P}$ , with a geometric Brownian motion

$$\frac{dV_t}{V_t} = \mu_V dt + \sigma_V dW_t,$$



where  $\mu_V$  and  $\sigma_V$  are the drift and volatility of the company value process, respectively (Grasselli and Hurd, 2010). The firm is said to have defaulted when the value of the assets is below that of the debt, that is, when the company is insolvent. In contrast to the original Merton credit model, it is assumed here that the value of the firm cannot be traded. This allows for the use of the real-world measure.

The equity-holders are entitled to the value of the firm less any debt owed by the company. But note that their limited liability protects them in the case that the company defaults. This structure allows the equity value at time  $T$  to be viewed as a European call option on the firm's assets,

$$E_T = \max(V_T - D, 0).$$

This model has the advantage that default events, at the company whose shares are being used as collateral, could be incorporated into the risk of the SBL — default events are expected to trigger a significant drop in the share price. The model will also naturally allow for the leverage effect, meaning that the volatility of the share price will increase as the asset value of the company approaches the debt value (Backwell *et al.*, 2016). This is expected to increase the probability that a stock in breach will end in default.

To employ this model, the dissertation will assume an initial debt-equity ratio of 0.2. A higher ratio will lead to a higher probability of default over the period in consideration. Though chosen arbitrarily, the choice of 0.2 is expected to provide reasonable probabilities of default. This ratio can be used to determine  $V_0$ , the initial asset level. The drift of the assets  $\mu_V$  and the volatility of assets  $\sigma_V$  will then still need to be determined. This can be done as follows.

Let  $E_t = f(V_t)$  be the equity value of the firm. Then, the call option price of the equity is

$$E_t = V_t \Phi(d_1) - D e^{-r(T-t)} \Phi(d_2),$$

where

$$d_1 = \frac{\ln \frac{V_t}{D} + (\mu_V + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V \sqrt{T-t}}$$

and

$$d_2 = d_1 - \sigma_V \sqrt{T-t}.$$

Using Ito's lemma,  $dE_t$  can be written as

$$\begin{aligned} dE_t &= \frac{\partial E_t}{\partial V_t} dV_t + \frac{1}{2} \frac{\partial^2 E_t}{\partial V_t^2} (dV_t)^2 \\ &= \mu_V V_t \frac{\partial E_t}{\partial V_t} dt + \sigma_V V_t \frac{\partial E_t}{\partial V_t} dW_t + \frac{1}{2} \sigma_V^2 V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} dt. \end{aligned}$$

Furthermore, assuming that  $E_t$  has GBM dynamics,

$$dE_t = \mu_E E_t dt + \sigma_E E_t dW_t,$$

and equating the volatility coefficients of both equations we get

$$\sigma_E E_t = \sigma_V V_t \frac{\partial E_t}{\partial V_t}.$$

Noting that  $\frac{\partial E_t}{\partial V_t} = \Phi(d_1)$ , the above equation can be used to solve for  $\sigma_V$  ([Grasselli and Hurd, 2010](#)).

The drift coefficient  $\mu_V$  can be obtained by setting

$$\begin{aligned} \mu_E E_t &= \mu_V V_t \frac{\partial E_t}{\partial V_t} + \frac{1}{2} \sigma_V^2 V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} \\ &= \mu_V V_t \Phi(d_1) + \frac{1}{2} \sigma_V^2 V_t^2 \frac{\phi(d_1)}{S_t \sigma_V (T - t)}. \end{aligned}$$

The parameters  $\mu_E$  and  $\sigma_E$  can be estimated from empirical data and used to set  $\mu_V$  and  $\sigma_V$  at initiation of the contract.

Note that if instead the estimated equity volatility is used as the asset volatility, the effect will be that the volatility of the simulated share price will be higher than would be suggested by reality.

### Rejected candidate models

A brief list of further models that were considered is provided here together with the rationale for why they were not implemented:

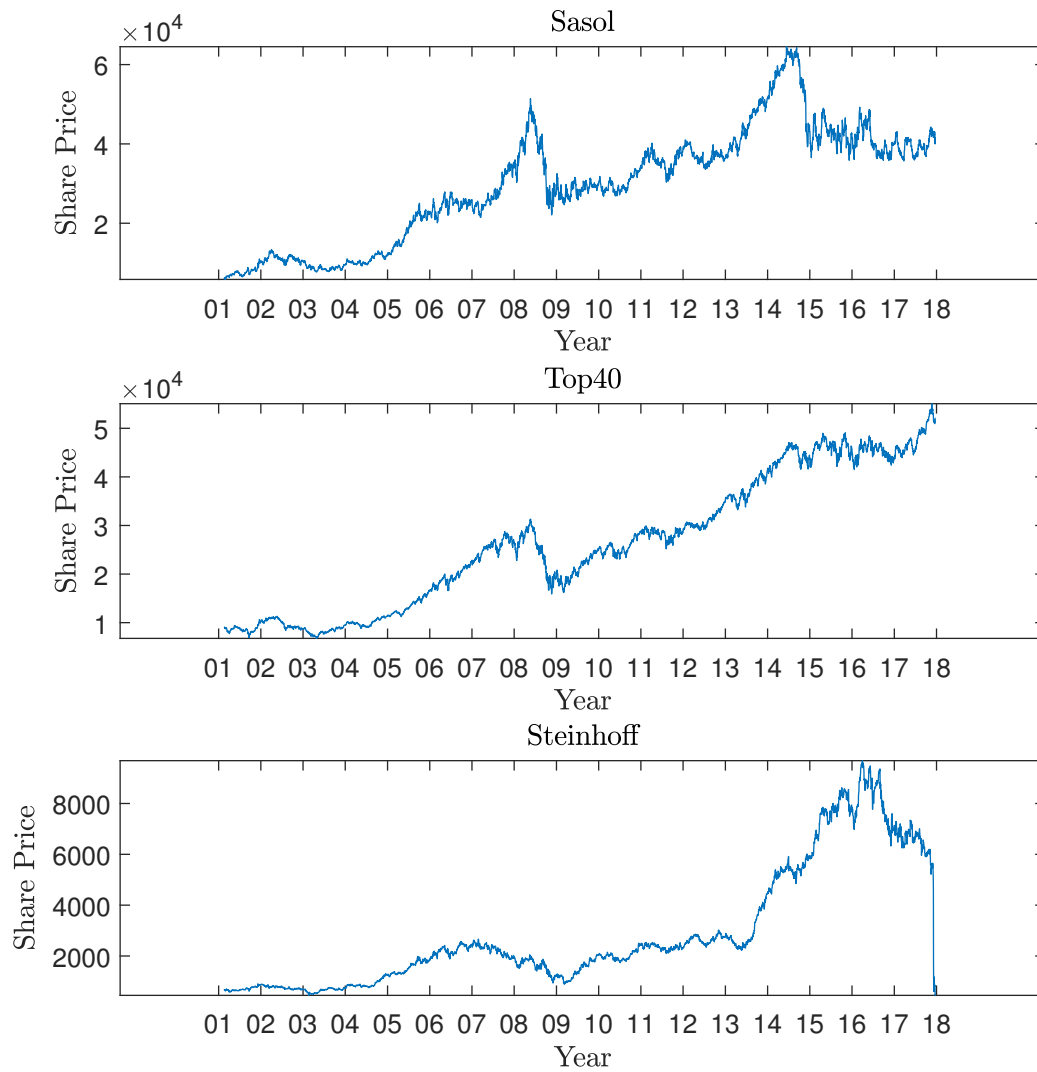
- Extensions of the Merton credit model, like the Black–Cox model and the Leland model, were also considered. The Black–Cox model allows for default prior to maturity (of the company whose shares are used as collateral, and not of the borrower). The Leland model adds more realism by introducing taxes and bankruptcy costs ([Sundaresan, 2013](#)). The primary advantage of the Merton credit model over these two models is its simplicity. Since the focus here is on obtaining realistic share price dynamics, the main aim in using the Merton credit model is to take advantage of the leverage effect it provides. It also has an endogenous probability of default like both the other models.
- Stochastic volatility models such as the Heston model provide for more realistic volatility behaviour and offer the ability to incorporate the leverage effect. This model is extended by the Bates model by allowing for jumps in the share price. However, the difficulty in using these models lies in estimating the necessary parameters, which is a problem outside the scope of this

dissertation. Nevertheless, a model such as the jumps plus model should be able to reasonably approximate the share price behaviour expected from these models.

### 3.3 Data

The daily closing prices of Sasol, Steinhoff and the JSE Top40 index were downloaded from Investing.com. The datasets are from the period from 16 February 2001 to 22 December 2017 and have about 4210 data points each. Sufficient share price history is required to capture the relevant features of the returns distribution including market crashes and bull runs.

These stocks were chosen for the following reasons. The Top40 would be representative of a diversified portfolio consisting of blue-chip stocks. Sasol is a share that has shown good growth in the past, with some periods of volatility that could affect the SBL. The drop in price suffered by Steinhoff would be an important example of the risk that the lender faces.



**Fig. 3.3:** The share price paths of Sasol, the Top40 and Steinhoff from 2001 to 2017.

### 3.4 Estimation of jump-diffusion parameters

The jump models used in the dissertation will need to be calibrated to the shares and portfolios being modelled. The calibration of jump models is made difficult by having to separate jumps from the Brownian motion increments (Backwell *et al.*, 2016).

Hanson and Westman (2002) derive the probability density of the Merton jump diffusion (3.1) and fit it to the empirical log-returns of the S&P500 Index, managing to obtain reasonable results. Their methodology is followed here to estimate the model parameters from the datasets described in Section 3.3.

Under  $\mathbb{P}$  the log-stock  $\log(S_t)$ , can be discretized with a time step  $\Delta t$  as

$$\Delta \log(S_t) = \mu_{ld}\Delta t + \sigma(W_{t+\Delta t} - W_t) + \sum_{i=1}^{N_{t+\Delta t} - N_t} \log(J_i),$$

where  $\Delta \log(S_t) = \log(S_{t+\Delta t}) - \log(S_t)$  and  $\mu_{ld} = \mu - \frac{1}{2}\sigma^2$ .

Note that the compound Poisson process,

$$\sum_{i=1}^{N_{t+\Delta t} - N_t} \log(J_i) = \sum_{i=1}^{N_{\Delta t}} \log(J_i),$$

has mean  $\lambda \Delta t \mathbb{E}[J_t] = \lambda \Delta t \alpha$  and variance  $\lambda \Delta t (\alpha^2 + \beta)$ .

Thus, the expectation and variance of the log-stock increment can be written as

$$\mu_{\Delta} = \mathbb{E}[\Delta \log(S_t)] = (\mu_{ld} + \lambda \alpha) \Delta t,$$

$$\sigma_{\Delta}^2 = \text{Var}[\Delta \log(S_t)] = \sigma^2 \Delta t + (\alpha^2 + \beta) \lambda \Delta t.$$

Hanson and Westman (2002) show that the density for the log-return differential of the log-normal jump-diffusion,  $d \log(S_t)$ , is given by

$$f_{d \log(S_t)}(x) = \sum_{k=0}^{\infty} p_k(\lambda dt) \phi(x; \mu_{ld} dt + \alpha k, \sigma^2 dt + \beta k),$$

and thus

$$f_{\Delta \log(S_t)}(x) \approx \sum_{k=0}^{\infty} p_k(\lambda \Delta t) \phi(x; \mu_{ld} \Delta t + \alpha k, \sigma^2 \Delta t + \beta k),$$

where  $p_k(\lambda dt)$  denotes the probability that a Poisson process with arrival intensity  $\lambda dt$  has  $k \in \mathbb{N}_0$  arrivals in the period  $dt$ , that is,

$$p_k(\lambda dt) = \frac{1}{k!} e^{-\lambda dt} (\lambda dt)^k.$$

The symbol  $\phi(x; a, b)$  denotes the normal density with mean  $a$  and variance  $b$  evaluated at  $x$ .

This density,  $f_{d\log(S_t)}(x)$ , has five parameters:  $\mu, \sigma^2, \lambda, \alpha$  and  $\beta$ . Denote the estimates of  $\mu_\Delta$  and  $\sigma_\Delta^2$  from the empirical log-returns as  $\hat{\mu}_\Delta$  and  $\hat{\sigma}_\Delta^2$ , respectively. The size of the parameter set can be reduced by setting the theoretical means and variances,  $\mu_\Delta$  and  $\sigma_\Delta^2$ , equal to the empirical means and variances,  $\hat{\mu}_\Delta$  and  $\hat{\sigma}_\Delta^2$ . This is done so as to imply the values of  $\mu_{ld} := \mu - \frac{1}{2}\sigma^2$  and  $\sigma^2$ ,

$$\begin{aligned}\mu_{ld} &= [\hat{\mu}_\Delta - \alpha\lambda\Delta t] / \Delta t, \\ \sigma^2 &= [\hat{\sigma}_\Delta^2 - \lambda\Delta t(\alpha^2 + \beta)] / \Delta t,\end{aligned}$$

thus simplifying the calibration procedure. Thus the approximate density for the discretized log-return differential can be written as

$$f_{\Delta\log(S_t)}(x) \approx \sum_{k=0}^{\infty} p_k(\lambda\Delta t) \phi(x; \hat{\mu}_\Delta - \alpha\lambda\Delta t + \alpha k, \hat{\sigma}_\Delta^2 - \lambda\Delta t(\alpha^2 + \beta) + \beta k).$$

The calibration procedure estimates the parameters that best fit the density to the empirical distribution of log-returns. This is done in the following way.

First, plot the histogram of the unconditional daily log-returns, and normalize it to have an area of one, to be representative of a probability distribution function. Then find the value of the histogram  $\hat{f}$  at each of the midpoints of the histogram bins. The theoretical distribution  $f$  is fitted to the empirical distribution  $\hat{f}$  by minimising the sum of squared errors, that is, the estimate of the parameter set  $\Theta$  is

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \sum_{i=1}^N \left( f(x_i; \Theta) - \hat{f}(x_i) \right)^2.$$

[Hanson and Westman \(2002\)](#) call this the histogram least squares method. The optimisation procedure was performed with the Matlab function `fminsearch`, which uses a derivative-free procedure. The jump diffusion parameters can also be estimated using maximum likelihood estimation.

The parameter set  $\{\lambda, -\alpha, \beta\}$  must have each of its elements constrained to be non-negative. The jump amplitude distribution is constrained to have a negative mean,  $\alpha$ , to better capture the negative skew of the log-returns ([Hanson and Westman, 2002](#)).

Since the estimation procedure relies on the shape of the histogram, it is important to consider the number of bins used. One would not want to use too many bins as that could result in overfitting, however, too few bins would result in failing to capture important features of the empirical returns distribution. [Hanson and Westman \(2002\)](#) use 50 bins for their estimates from the 1657 data points from the daily

**Tab. 3.1:** Results of estimation procedure for the Sasol, Steinhoff and the Top40 datasets. The results are displayed for the cases where 50, 100, 200, 300 and 400 bins were used in the histogram least squares method to show the sensitivity of the estimates to the number of bins.

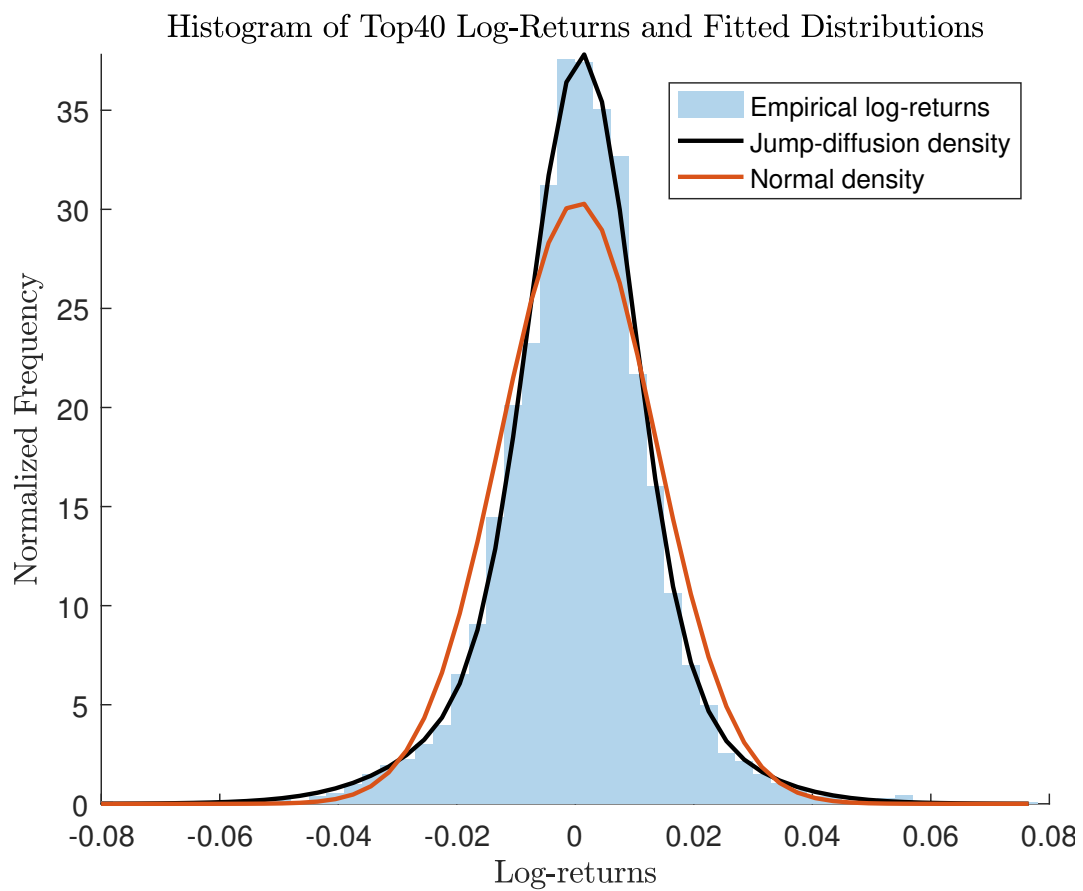
Share	Bins	$\mu$	$\sigma$	$\lambda$	$\alpha$	$\beta$
Sasol	50	0.260	0.153	326.0	-0.00047	0.00027
	100	0.248	0.164	280.5	-0.00052	0.00030
	200	0.231	0.165	280.8	-0.00046	0.00030
	300	0.214	0.164	282.5	-0.00039	0.00030
	400	0.219	0.164	281.1	-0.00041	0.00030
Steinhoff	50	1.794	0.036	715.2	-0.00254	0.00031
	100	-0.047	0.238	39.9	-0.00000	0.00426
	200	-0.043	0.220	51.3	-0.00000	0.00347
	300	-0.041	0.212	57.6	-0.00000	0.00315
	400	-0.041	0.212	58.3	-0.00000	0.00312
Top40	50	0.317	0.133	115.6	-0.00193	0.00021
	100	0.316	0.128	134.8	-0.00164	0.00019
	200	0.304	0.128	130.2	-0.00160	0.00020
	300	0.308	0.128	130.8	-0.00163	0.00020
	400	0.301	0.129	126.7	-0.00163	0.00020

closing price of the S&P500 dataset. This dissertation considers varying the number of bins used to generate the histogram as a rudimentary check on the robustness of the estimation procedure. The results from this analysis are displayed in Table 3.1.

Since Matlab selects the bin edges to cover the range of the data, if there are returns that are significant outliers, the bins will cover a larger range and the histogram will not capture the relevant details. Thus, to avoid poor results, the number of bins used in such cases will need to be increased. See Appendix A.1 for a figure illustrating the error for the case with 50 bins.

From Table 3.1 it would seem that using anywhere between 200 and 400 bins would provide reasonable estimates. This dissertation will use 300 bins.

The jump-diffusion density with the estimated parameters is plotted along with the empirical log-returns and the fitted normal density in Figure 3.4. The jump-diffusion density appears to give a good fit. A kurtosis larger than that of the normal density, and a negative skew can also be observed in the figure.



**Fig. 3.4:** Histogram of log-returns and fitted densities for the Top40 dataset. The number of bins used here was 50.



## Chapter 4

# Results

### 4.1 Credit measures

The results of the investigation for all the models considered in Chapter 3 are presented in this chapter. To give a more complete picture of the results and how they would differ for different sets of parameters, the results are presented for the parameter sets estimated from the Sasol, Steinhoff and Top40 datasets. These sets of parameters are given in Table 4.1.

A Monte Carlo simulation was implemented in Matlab where the number of simulations was set at 200 000. The loan being considered has a principal of R100 million with a maturity of 3 years. The initial coverage ratio is set at 2.5, and the minimum coverage ratio at 1.5. The risk-free rate is assumed to be 6% with the spread being 4%. Upon breach, the stock is assumed to have a 10 day liquidation period, with a downward liquidity pressure of 1% for each day in the liquidation period. The stock is assumed to have a dividend yield of 1%.

The one-year probability of a jump to distress is set at 0.3%, and the probability of a jump to empirical distress is assumed to be 1%. The debt-equity ratio for the Merton credit model is assumed to be 0.2.

**Tab. 4.1:** Model parameter estimates for each of the three datasets.

Stock	$\mu$	$\sigma$	$\mu_j$	$\sigma_j$	$\lambda$	$\alpha$	$\beta$
Sasol	0.1167	0.3349	0.2138	0.1638	282.5	-0.0004	0.0174
Steinhoff	-0.0187	0.4759	-0.0412	0.2124	57.6	-0.0000	0.0561
Top40	0.1033	0.2065	0.3078	0.1283	130.8	-0.0016	0.0141

For each of the datasets, the probability of breach (PB), the probability of loan breach (PLB) and the probability of default (PD) are calculated and given in percentage terms. The loss given breach (LGB), loss given default (LGD) and expected

**Tab. 4.2:** Estimates of the credit measures using the parameters estimated from Sasol shares.

Model	PB (%)	PLB (%)	PD (%)	LGB (k)	LGD (k)	EL (k)	ENPV (m)
GBM	41.79	0.00	0.00	0.00	0.00	0.00	9.64
GBM (JTD)	42.20	0.35	0.33	429.03	54132.16	181.07	9.43
GBM (JTE)	43.34	1.10	0.23	49.98	9478.28	21.66	9.51
Empirical	41.51	0.00	0.00	0.00	0.00	0.00	9.66
Merton Credit	35.51	0.00	0.00	0.00	0.00	0.00	10.11
MJD	36.98	0.00	0.00	0.00	0.00	0.00	9.93
MJD+	36.98	0.17	0.00	0.00	0.00	0.00	9.93
MJD+ (JTD)	37.37	0.57	0.36	526.52	53978.00	196.75	9.72

loss (EL) in thousands of Rands and the expected NPV (ENPV) in millions of Rands are given as well. The abbreviations JTD, JTE and MJD stand for jump to distress, jump to empirical distress returns, and Merton jump diffusion, respectively. The abbreviation MJD+ refers to the jumps plus model.

The results of the simulation involving parameters estimated from the Sasol dataset are presented in Table 4.2. It is only the jump to distress models and the JTE model that lead to any material losses upon default. Other than these, only the jumps plus model gives a non-zero probability of loan breach. The jump to distress term results in an LGD of above R50 million on average while the jump to empirical distress gives a value of about R9.5 million. The main reason for this difference is that large downward jumps are relatively infrequent in the empirical distress distribution. Another contributing factor is that, in the JTE model, a smaller proportion of stocks that breach result in default on the SBL.

The results where the parameters were estimated using Steinhoff shares (see Table 4.3) show large PB values and non-zero probabilities of loan breach for all the models. It is only the GBM model and the Merton credit model that fail to give non-zero probabilities of default. The empirical model gives a PD value that is unrealistically high. This is a result of sampling log-returns repeatedly from the same empirical distribution. Given that only a small proportion of the shares in the market would be expected to suffer the plunge in share price that Steinhoff experienced, this should be seen as an overestimate of the results expected on the typical share.

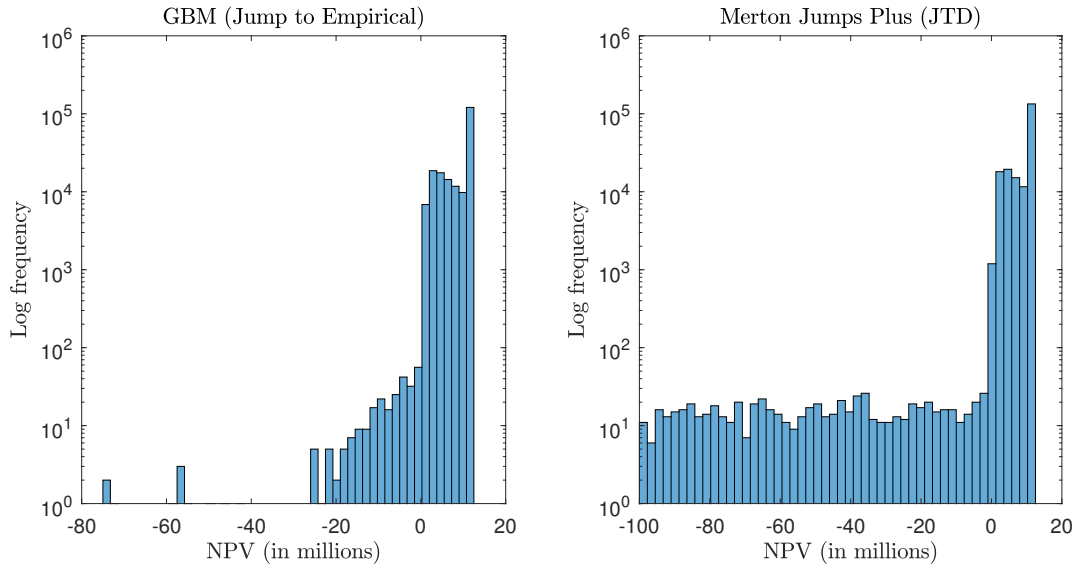
In Table 4.4, results are provided for the models with parameters estimated from the Top40 index. Given that the Top40 is a more diversified portfolio, it would make

**Tab. 4.3:** Estimates of the credit measures using the parameters estimated from Steinhoff shares.

Model	PB (%)	PLB (%)	PD (%)	LGB (k)	LGD (k)	EL (k)	ENPV (m)
GBM	77.04	0.18	0.00	0.00	0.00	0.00	6.23
GBM (JTD)	77.42	0.41	0.22	151.95	54593.76	117.65	6.10
GBM (JTE)	77.77	1.21	0.19	21.61	8823.27	16.81	6.16
Empirical	71.13	11.25	8.81	2284.15	18442.50	1624.78	5.61
Merton Credit	75.21	0.20	0.00	0.00	0.00	0.00	6.51
MJD	69.56	0.47	0.01	0.24	2520.96	0.16	6.94
MJD+	69.56	3.06	0.08	4.20	3584.43	2.92	6.94
MJD+ (JTD)	69.63	3.28	0.33	192.28	40757.35	133.89	6.80

**Tab. 4.4:** Estimates of the credit measures using the parameters estimated from the Top40 index.

Model	PB (%)	PLB (%)	PD (%)	LGB (k)	LGD (k)	EL (k)	ENPV (m)
GBM	16.28	0.00	0.00	0.00	0.00	0.00	11.61
GBM (JTD)	17.01	0.46	0.44	1397.28	54205.67	237.69	11.34
GBM (JTE)	18.59	1.08	0.24	132.46	10153.36	24.62	11.44
Empirical	16.20	0.00	0.00	0.00	0.00	0.00	11.61
Merton Credit	11.98	0.00	0.00	0.00	0.00	0.00	11.81
MJD	14.80	0.00	0.00	0.00	0.00	0.00	11.67
MJD+	14.80	0.00	0.00	0.00	0.00	0.00	11.67
MJD+ (JTD)	15.57	0.48	0.46	1670.44	57149.23	260.03	11.38



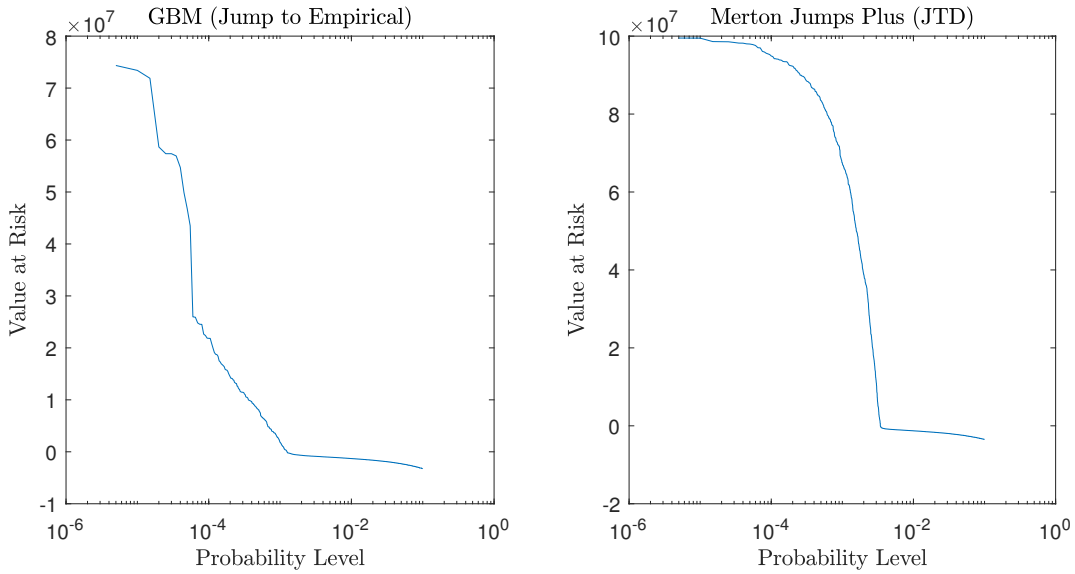
**Fig. 4.1:** The NPVs for the SBL where the models used are the GBM with jump to empirical distress distribution and the jumps plus model with JTD. The model parameters are estimated from the Sasol dataset. Note that a log-scale is used for the frequency axis.

sense that it has the lowest probabilities of breach amongst the three datasets. As for the results from the Sasol parameter set, it is only the models that had a jump to distress or a jump to empirical distress that had any significant probabilities of default. Here, however, the PDs and LGDs are possibly too high, relative to the single-stock case of Sasol, given the diversified nature of the index. Thus, for such a portfolio, it would be sensible to choose a lower probability for the event that a jump to (empirical) distress occurs.

To better understand the results, the distributions of the NPVs under the jump to empirical and jumps plus with JTD models are plotted in Figure 4.1. The JTE model has few large losses relative to the jumps plus model with JTD. This is a consequence of the relative infrequency of significant drops observed in the share prices used in the empirical distribution, and can explain the differences between the credit measures obtained for these two models.

In the histogram of the jumps plus model with JTD, one would desire the frequency of losses to taper off as the losses become more extreme. The choice of a uniform distribution for the fraction  $R$ , which determines to what price the share price will jump, could be the reason that the desired tapering off does not occur. An empirically justified distribution for  $R$  could instead be chosen.

Typically, capital requirements are determined using value at risk. Figure 4.2



**Fig. 4.2:** The Value at Risk for different probability levels for an SBL where the models used are the GBM with jump to empirical distress distribution and the jumps plus model with JTD. The model parameters are estimated from the Sasol dataset.

shows the value at risk of the SBL with varying confidence levels over a 3-year horizon for the GBM with JTE and jumps plus with JTD models. The jumps plus JTD model, at the lowest probability level considered, allows for the scenario where the lender can lose all his capital, unlike the GBM with JTE model. The reason for this is explained in Figure 4.1, where the largest observed loss for the jump to empirical model is just less than R80 million.

## 4.2 Sensitivities to the SBL parameters

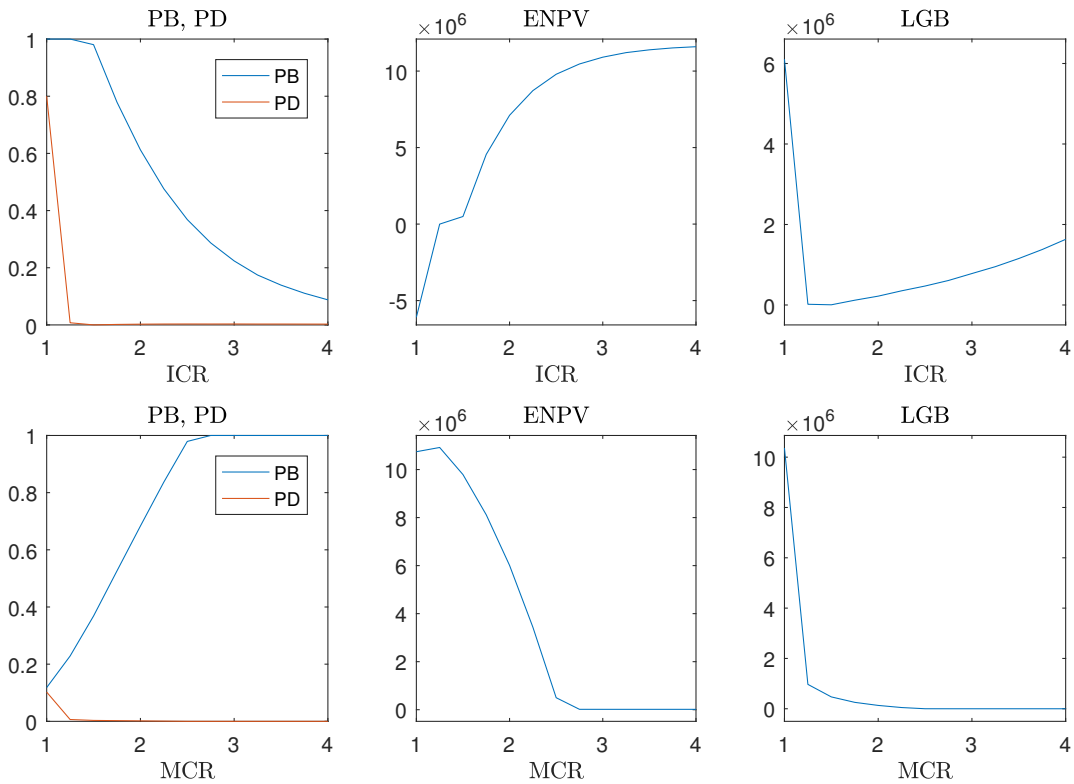
The results of the investigation above are dependent on the values selected for the SBL parameters and hence this section considers the changes that would occur in the main results for changes in these parameters. The parameters are changed in isolation to better understand their contribution to the SBL's risk characteristics. The other parameters are kept at their original level. The sensitivities are performed for the Sasol dataset.

The effect of changing the parameters in the jumps plus model with JTD is presented in this section. The results for the GBM with JTE model can be found in Appendix A.

The initial and minimum share coverage ratios are key determinants of the at-

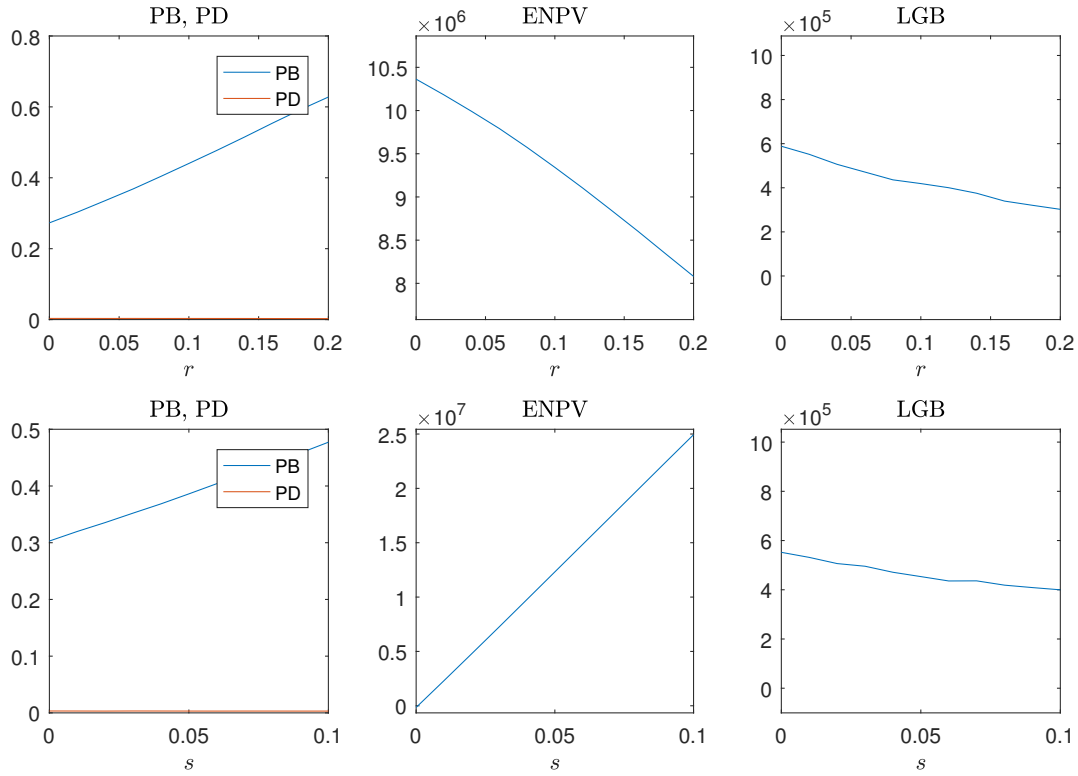
tractiveness of the SBL to the borrower, and of the profitability of the contract to the lender. Figure 4.3 shows the effect of changing ICR on the PB, PD, ENPV and LGB of the contract. As would be expected, the probabilities of breach and default both decrease with an increase in the initial share coverage ratio, with the PD value decreasing the most rapidly. When the ICR is between 1 and 1.5, the ENPV figure indicates that the SBL is not profitable. Increasing the ICR to 3 results in an expected net profit of around R10 million. Further ICR increases beyond this level do not contribute much to the ENPV for the SBL under this model. The lender will, however, need to choose a sufficiently low ICR to attract customers.

The MCR behaves somewhat differently from the ICR as increasing the MCR increases the probability of breach. The increase in MCR also lowers the ENPV of the SBL. Note here that the default ICR is set at 2.5, so any values of MCR above 2.5 will lead to meaningless results.



**Fig. 4.3:** The effect on the credit measures of changes in the initial and minimum share coverage ratios for the jumps plus model with JTD.

An increase in the risk-free rate  $r$  leads to an increase in the PB figure (shown in Figure 4.4). A higher rate of interest means that the breach barrier rises at a faster rate (since the outstanding loan value increases at the new higher rate) and thus

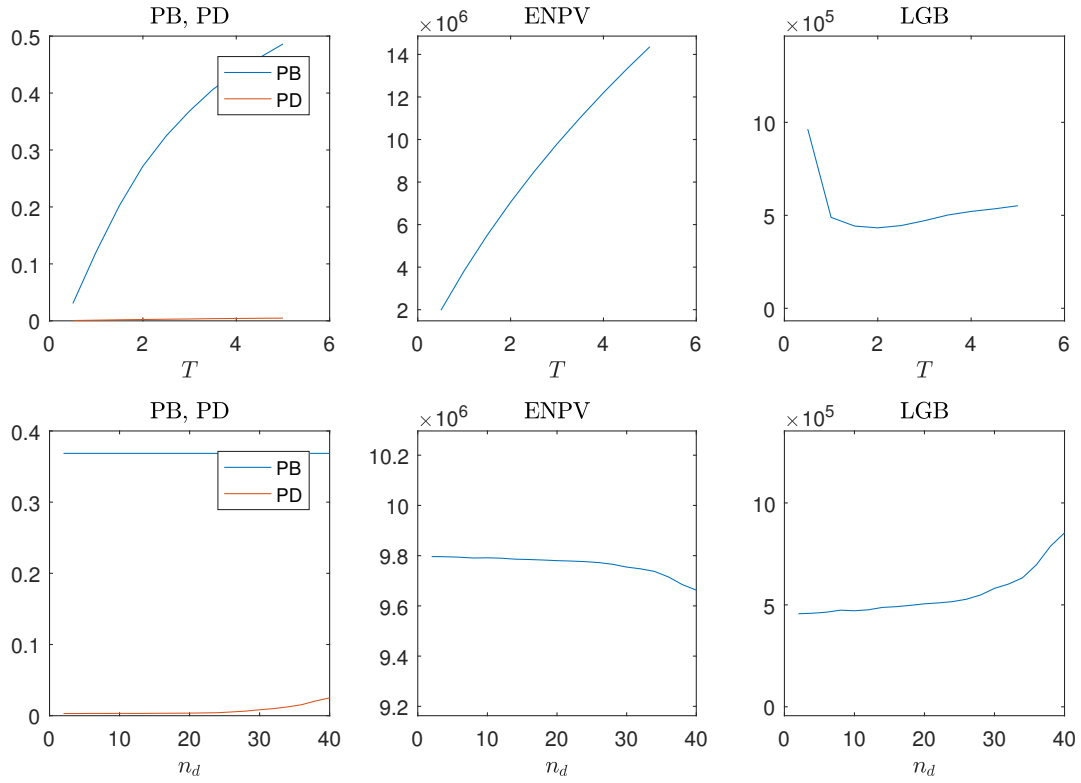


**Fig. 4.4:** The effect on the credit measures of changes in the risk-free rate and the spread for the jumps plus model with JTD.

breaches are more likely. The ENPV has a negative relationship with the interest rate since it is used for discounting the future cashflows. There are at least two reasons why the LGB will decrease with a higher interest rate. Since breaches (that are not a result of a JTD) are expected to occur earlier as a result of the higher breach barrier, the probability that a jump to distress can now occur over the lifetime of the contract is less likely, and the LGB of the contract will thus decrease. Also, earlier breaches could mean that the loan values are smaller on average, thus leading to lower losses on breach.

Raising the spread  $s$  increases the PB value as the spread leads to a higher loan value over time, which effectively determines the barrier at which breach occurs. A higher spread will result in a higher ENPV as would be expected. It will also lead to a lower LGB for the same reasons as for a higher interest rate  $r$ .

A longer term to maturity results in a larger ENPV as the loan is able to accumulate at the rate of  $r + s$  over a longer period of time. This also will mean that the probabilities of breach occurring will be higher given the longer period of time available for a drop in the price. The LGB is relatively high for a contract with a short maturity. This is since the breaches that occur so early in the contract would



**Fig. 4.5:** The effect on the credit measures of changes in the maturity of the SBL and the number of days taken to liquidate for the jumps plus model with JTD.

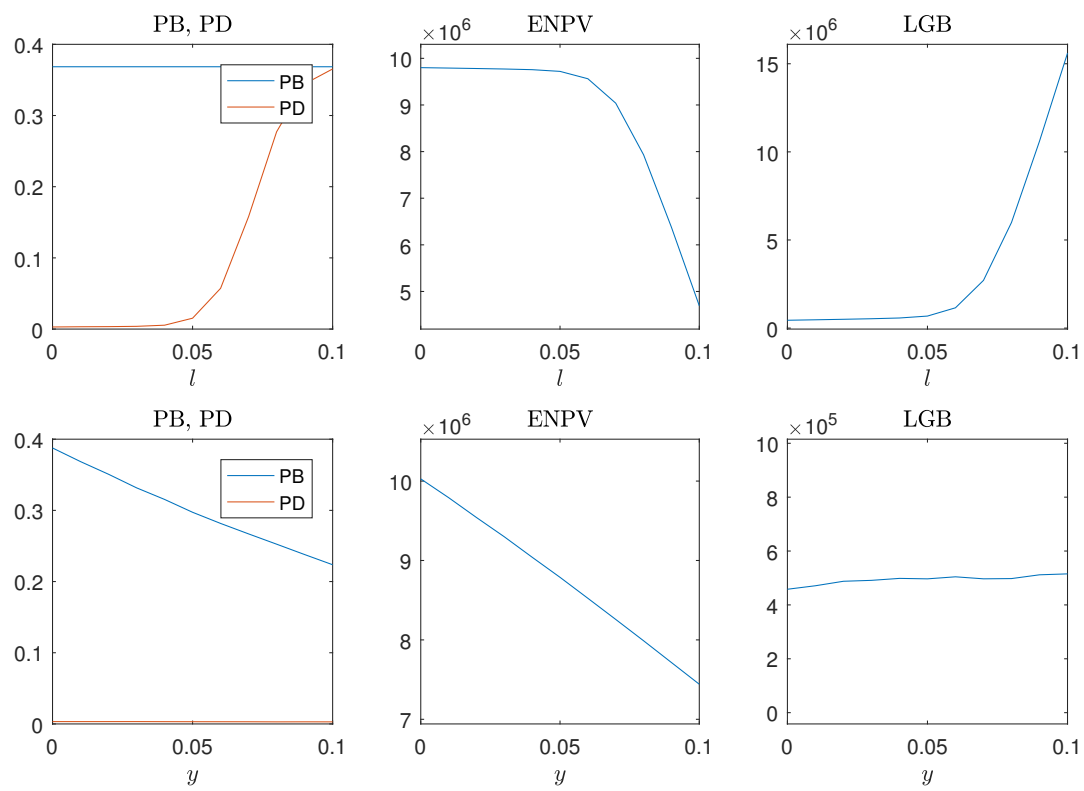
most likely be a result of a jump to distress, which would typically lead to a large loss.

Changing the number of days taken to liquidate the shares,  $n_d$ , has no effect on the probability of breach, as should be expected. The number of paths resulting in default increases as more days are taken to liquidate leading to higher LGBs and lower ENPVs.

The liquidity pressure variable affects the proportion of breaches that result in default by forcing down the price. This leads to a decrease in the ENPV and an increase in the LGB value (see Figure 4.6).

Increasing the dividend yield parameter,  $y$ , has a negative effect on the PB measure. Every dividend payout reduces the share price and the loan value by an equal amount, but since the breach barrier is a multiple of the loan amount ( $MCR \times N_t$ ), it will reduce by a larger amount. With a large increase in  $y$ , a larger portion of the loan is repaid early and the contract is not as profitable.





**Fig. 4.6:** The effect on the credit measures of changes in the liquidity pressure variable and the dividend yield for the jumps plus model with JTD.

## Chapter 5

# Conclusion

SBLs provide significant benefits to both the borrower and the lender. The stock-collateral provides mitigation of the credit risk of the borrower, and thus makes the SBL arrangement more attractive to the lender by improving the credit characteristics of the loan, allowing the borrower to pay a lower rate of interest.

The probability of default determines the likelihood of the lender suffering losses. To model the SBL to get realistic measures of credit risk, share-price models are considered that are expected to provide an adequate probability of default.

Overall, from the analysis conducted in this dissertation, SBLs seem to be without too much financial risk to the lender. The use of appropriate initial and minimum coverage ratios seems to provide adequate protection to the lender, and it is only in rare cases that the lender will suffer a loss. However, it is possible that these losses can be large when they do occur.

The investigation also considers varying the SBL parameters to assess how these affect its credit characteristics. The prudent selection of some parameters (like the ICR and MCR) can allow the lender to determine the extent to which he takes on credit risk, and the competitiveness of the contract.

### 5.1 Discussion on capital requirements

Lenders will be required to hold capital against the risk of default. The higher the risk of default and the associated loss given default, the more capital the lender will have to hold. Since this capital comes at a cost to the lender, the lender will wish to reduce the amount of capital held as much as possible.

In the analysis of the SBLs conducted in this dissertation, it would appear that the risk of default is not significantly large. This would seem to suggest that the lender would not need to hold much capital to back these contracts. Now, for example, if lenders are currently holding a large proportion of the nominal as capital, say about 80%, then by reducing the amount held to even about 50%, the lender

will be able to realize large savings on its capital charge. A proportion of these savings could be transferred to the borrower, and thus be used to determine SBL spreads at more attractive rates.

## 5.2 Possible further research

One shortcoming of this dissertation is that data on actual SBLs sold in the market were not available. If this were available, the results of the investigation could be compared with the actual frequency of loan breaches, defaults, and the typical magnitudes of NPVs and LGDs. Thus, the analysis has been kept general enough that its applicability is as broad as possible.

Some aspects of the dissertation could be developed further with more research. For example, the distribution of the fraction to which jump to distress occurs is assumed to be uniform. This might not be the best choice of distribution — distributions that are empirically justified could be researched. Also, more datasets could be aggregated to create the empirical distress distribution.

To address the concern that it is equally likely for a well-performing share to abruptly jump to distress as it is for a poorly performing share, a jump to distress hazard rate that is dependent on the share's price history could be considered. Here, the probability the share will jump to distress would increase if the share has done poorly over a certain period of time. So in effect, a share that has resulted in breach will have a higher probability of jumping to distress than if it had not breached.

Furthermore, sufficiently capturing the risks of liquidation under distress is an important consideration given its impact on the risk and profitability of the contract. Thus, having the liquidity pressure term dependent on the volumes traded and other share price information could add more realism to the modelling.

Share prices dependent on a stochastic interest rate will allow for some dependence by the share on the interest rate.

Even though the dissertation concentrates mainly on single-stock portfolios, the use of multi-stock portfolios, that are not necessarily well-diversified, could be explored. The correlations between stocks in the portfolio would then have to be considered.

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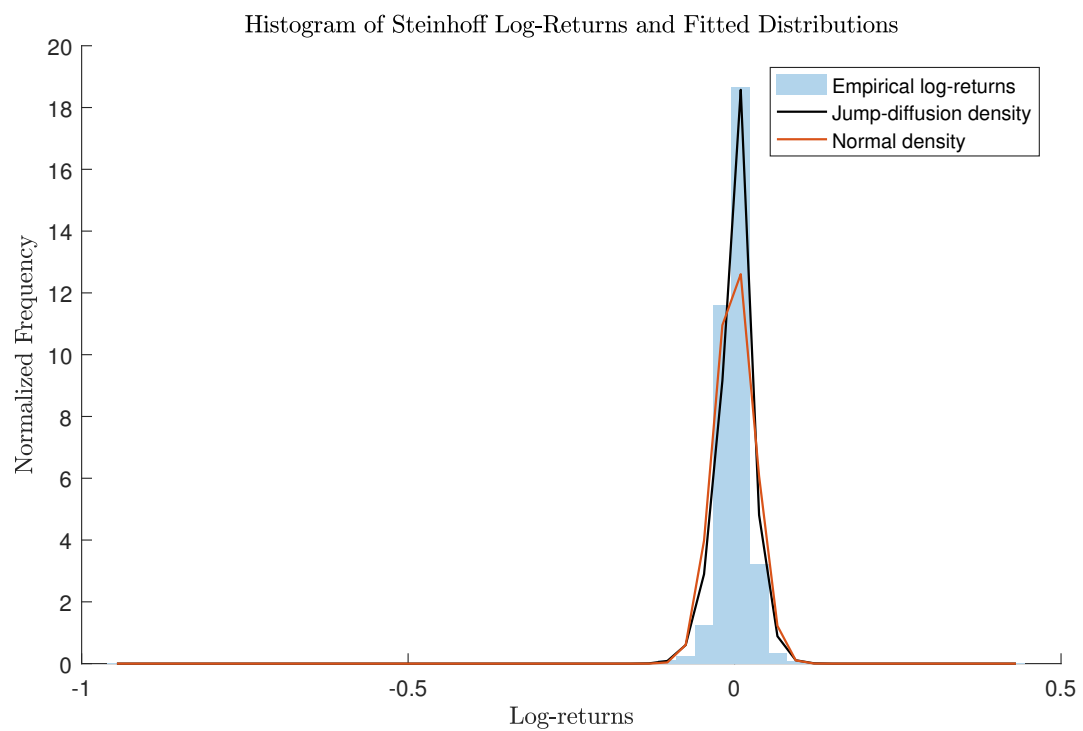
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## Appendix A

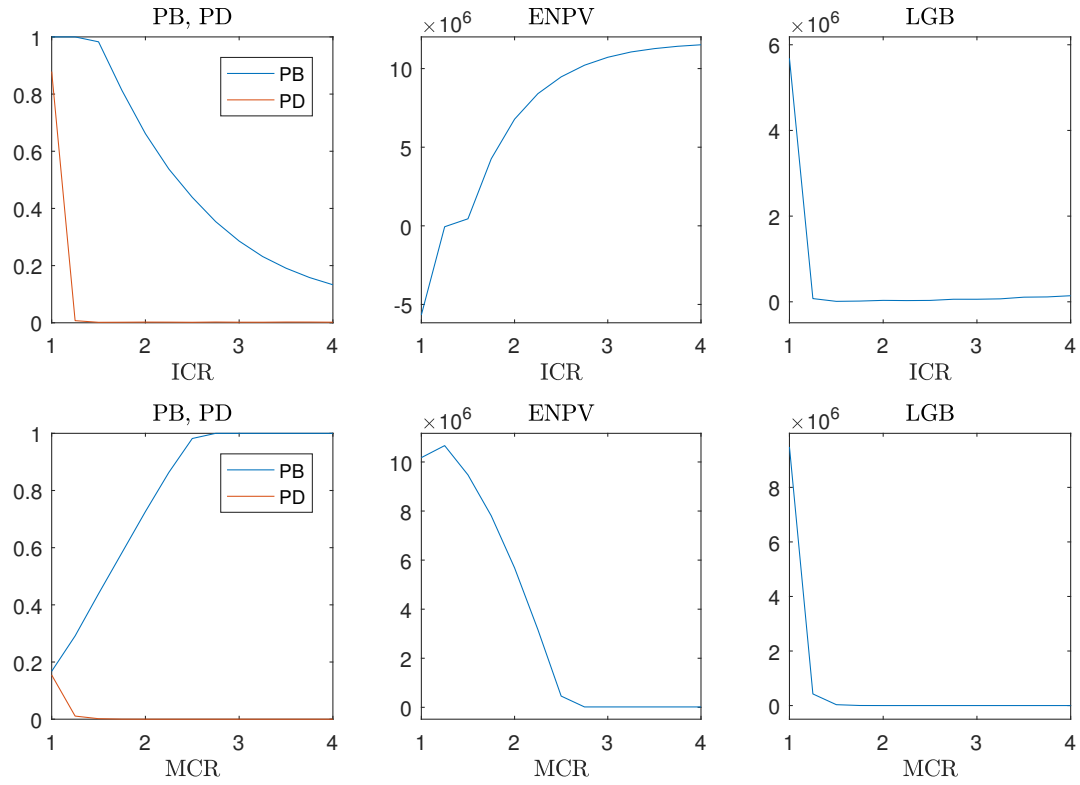
# Appendix

### A.1 Calibration

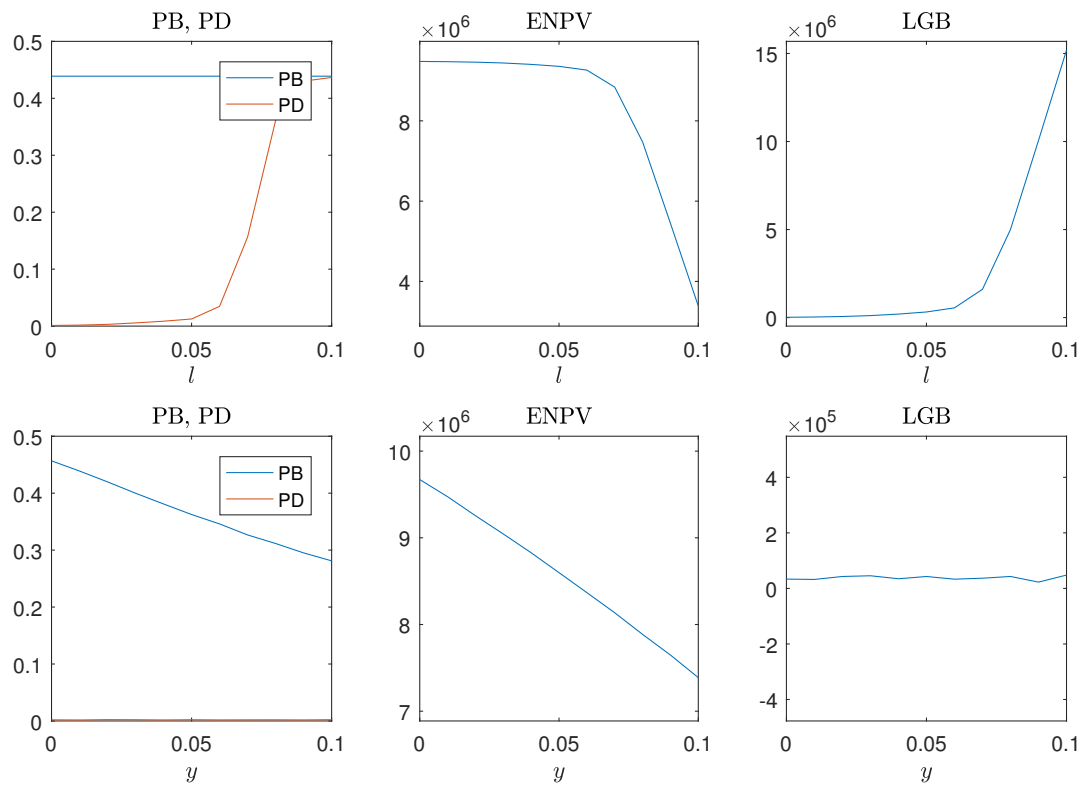


**Fig. A.1:** Histogram of log-returns and fitted densities for the Steinhoff dataset. The number of bins used here was 50.

## A.2 GBM with jump to empirical distress distribution

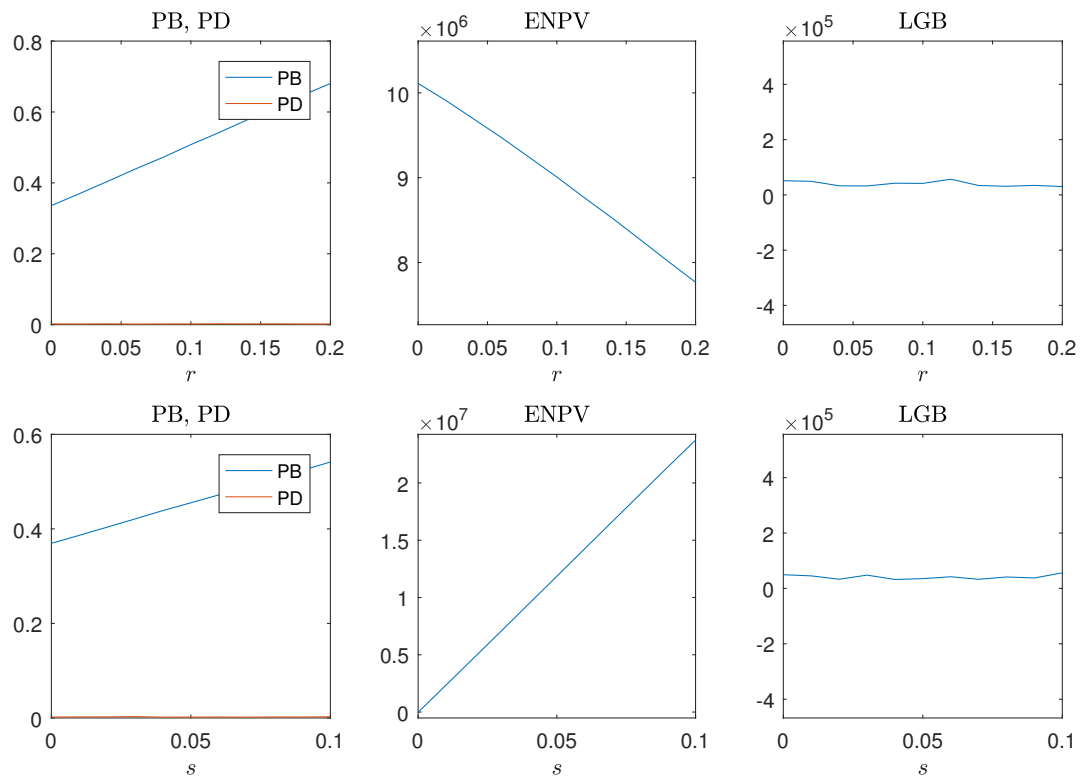


**Fig. A.2:** The effect on the credit measures of changes in the initial share coverage ratio and the minimum share coverage ratio for the GBM with jump to empirical distress model.

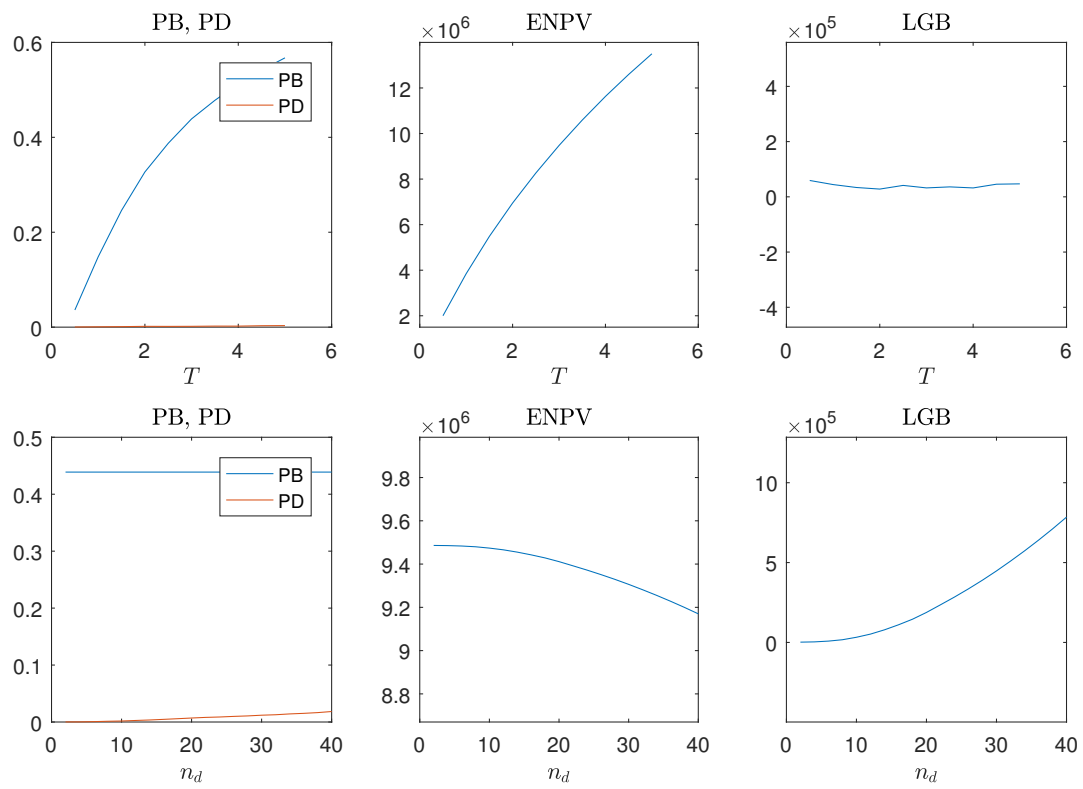


**Fig. A.3:** The effect on the credit measures of changes in the liquidity pressure variable and the dividend yield for the GBM with jump to empirical distress model.





**Fig. A.4:** The effect on the credit measures of changes in the risk-free rate and the spread for the GBM with jump to empirical distress model.



**Fig. A.5:** The effect on the credit measures of changes in the maturity of the SBL and the number of days taken to liquidate for the GBM with jump to empirical distress model.